

## Auction Models with Resource Pooling in Supply Chain Management

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**Abstract:** Supply chain management (SCM) has attracted much attention nowadays, aiming to reduce total operational cost while implementing quick response. Information sharing and cooperation mechanism implemented in traditional SCM may help achieve that but usually with many obstacles in practice, such as the difficulties in the collection of proper partners, in the allocation of the total supply chain cost and in the determination of the contract price, etc. However, auction mechanism introduced in SCM can overcome the information asymmetry between manufacturers and suppliers, which celebrates three properties: incentive compatibility, pricing balance and economic efficiency for both sides. In this paper, we propose two theoretic models of supply chain auctions with resource pooling according to the Vickrey auction principle. The two models are: supply auction model with demand resource pooling, and demand auction model with supply resource pooling. And for the proposed auction models, we present two corresponding algorithms to allocate resources in the auction process by linear programming. Furthermore, we study the incentive compatibility and define the Walrasian equilibriums for the proposed auction models. We show that the solutions of the proposed algorithms are Walrasian equilibriums.

**Keywords:** SCM, Resource Pooling, Vickrey Auction Mechanism, Walrasian equilibriums

### I. Introduction

Supply chain management (SCM) aims at improving the overall performance of all the enterprises in the supply chain as well as the supply chain as a whole. It integrates inside and outside enterprise resources to meet the quick change of customers' needs and improves the competitive edge of the supply chain. Supply chain considers all business processes from point of origin to point of consumption, involving material procurement, product research & development, manufacture, marketing, logistics and after-sale services [1, 2], etc.

To reduce operational cost and to respond quickly in SCM, three key factors should be considered: the supply chain network construction, the information sharing mechanism and the cooperation mechanism. However, the implementation of the three factors in supply chain invariably meet many problems in practices, such as what to

cooperate for, who to cooperate with and how to cooperate etc. So it is necessary to introduce an efficient coordination mechanism in SCM to solve the problems.

In highly competitive business environment, we can naturally expect participants in the supply chain to be motivated solely by self-interests, which often results in inefficiencies for the supply chain as a whole. Therefore, we could exploit some auction mechanisms to make the process more efficient and profitable, improving the well-being of all participants therein. Auction is a market mechanism where the buyers and seller agree on the items of interest and on the payment and delivery conditions [3, 4]. There are three participants in an auction: the buyer, the seller, and the auctioneer. Usually, the auctioneer only plays the role as an agency to organize the auction. There are four basic types of auctions widely used and analyzed: the ascending-bid auction (also called the open, oral, or English auction), the descending-bid auction (used in the sale of flowers in the Netherlands and so also called the Dutch auction by economists), the first-price sealed-bid auction, and the second-price sealed-bid auction (the latter is also known as the Vickrey auction by economists)[5]. Besides, game theoretic and combinatorial ideas [6, 7] have been widely applied in the design of algorithms and mechanisms for the various auction models, such as VCG mechanism [5], single-minded auction [8], combinatorial auction [9], online auction [10], computing grid auction [11], digital goods auction [12], market equilibrium [13] and Walrasian equilibrium [14].

In the paper, by introducing the auction mechanism, we aim to realize the following three functions: (1) Price mining. Auction is an open mechanism with multiple participants. Every bid is based on its assessment on the targeted products or services. Everyone has his own utility and purchasing ability. So, the final price reflects the market value of the auction. (2) Dynamic supply chain construction. Through auction, firm can find appropriate partner efficiently. Most of the time, auctions are one-time transactions. But if two firms, matched through auction, cooperate well in supply chain, then they can continue this relationship. If one of parties is dissatisfactory, then a new auction can be held to find a new partner. (3) Resources integration. In business practices, especially in supply chain auctions, auctioneers can integrate resources and ally buyers or sellers to gain more efficiency in auctions.

In this paper, our main focus is on supply and demand auction systems with resource pooling in supply chain from a theoretical modeling perspective. The supply and demand auction systems in supply chains among manufacturers and suppliers serve as information sharing mechanisms. The main purpose is to match the supply and demand such that a supply chain can achieve incentive compatibility and economic efficiency. A key feature of an ideal auction system is that it provides access to a rich set of information services and resource integration. We design such a supply and demand auction system that can integrate resources to efficiently match the supply and demand. The resources to be auctioned may be distributed in different locations or spots with connected paths, which we call a resource pooling network. Participating auctioneer desires to integrate the various resources to discover auction price efficiently although they also need to take into consideration some extra fees such as logistic costs besides the price.

The paper is organized as follows. In section 2, we propose two theoretic models of supply chain auctions with resource pooling according to the Vickrey auction principle. They are supply auction model with demand resource pooling, and demand auction model with supply resource pooling. In section 3, for the proposed auction models, we present two corresponding algorithms to allocate resources in the auction process by linear programming. In section 4, we study the incentive compatibility and define the Walrasian equilibriums for the proposed auction models. We show that the solutions of the proposed algorithms are Walrasian equilibriums. Finally, we conclude our paper with remarks and future research directions in section 5.

## II. Auction Models with Resource pooling

We consider two auction models with resource pooling: a demand auction model and a supply auction model. In those models we introduce an auctioneer, who shall organize the auction and implement the mechanism in place. The proposed auction models can be utilized for different situations, but they share one common objective, that economic efficiency can be improved under the condition of incentive compatibility, and from information collection and resource integration. The demand auction model is utilized for the market where the supply is much larger than demand. In such a market, the auctioneer collects all the demand from the manufacturers and conducts a reverse auction with the suppliers. Usually the more demand the auctioneer collects, the more commission from manufacturers and more discount from suppliers. Conversely, the supply auction model suits for the situation where the supply is short and there is serious information asymmetry between sellers and buyers. Finally, the double auction model fits for the market where both supply and demand situations are unknown. To facilitate further discussion, we first make some assumptions, give some definitions and explain some general notations.

**Assumptions 2.1** There is some kind of information asymmetry between manufactures and suppliers, which means that manufactures do not know any information about the suppliers and the suppliers also can not estimate how big the production's market is.

**Assumptions 2.2** Supply and demand bidding processes are not open between manufactures and suppliers;

**Assumptions 2.3** The auctioneer gets its profit not only from auction service commission but also from resource integration and information collection, e.g. demands collection and supply resource control etc.

### Definition 2.1 (Demand Resource Pooling Network)

Suppose that there exist  $m'$  manufacturers. Each of them has commercial alliance relationship with the auctioneer. Let the set of the  $m'$  manufacturers be expressed by  $M' = \{M'_1, \dots, M'_{m'}\}$ . We define the manufacturers network  $G_{M'} = (M', E'_1, w'_1)$  as the demand resource pooling network, where  $E'_1$  is the set of the edge  $e$  with two end nodes  $M'_i \in M'$  and  $M'_j \in M'$ .  $(M'_i, M'_j) \in E'_1$  means that it is transportable between the two end nodes  $M'_i$  and  $M'_j$ ;  $w'_1$  is weights defined on every edge  $e \in E'_1$  as the logistic fee per unit goods between the two ends of  $e$ . If  $(M'_i, M'_j) \notin E'_1$ , then define  $w'_1(M'_i, M'_j) = \infty$ . Denote  $w'_1(M'_i, M'_j)$  be the lowest logistic cost between  $M'_i$  and  $M'_j$ , which can be calculated by,

$$w'_1(M'_i, M'_j) = \min \left\{ \sum_{e \in \Gamma} w'_1(e) \mid \Gamma \text{ is a path between } M'_i \text{ and } M'_j \in M' \right\} \quad (1)$$

**Remark 2.1** The commercial alliance relationship between the auctioneer and each manufacturer means that the auctioneer can collect demand from every manufacturer in, make some kind of demand pricing recommendation for them, and integrate the demands for auction as a whole on behalf of them. In return, each manufacturer in will pay a commission fee to the auctioneer in proportion to the purchase amount.

Similarly, we define the supply resource pooling network as follows.

### Definition 2.2 (Supply Resource Pooling Network)

Suppose that there exist  $n'$  suppliers. Each of them has commercial alliance relationship with the auctioneer. Let the set of the  $n'$  suppliers be expressed by  $S' = \{S'_1, \dots, S'_{n'}\}$ . We define the suppliers network  $G_{S'} = (S', E'_2, w'_2)$  as the supply resource pooling network, where  $E'_2$  is the set of the edge  $e$

with two end nodes  $S'_i \in S'$  and  $S'_j \in S'$ .  $(S'_i, S'_j) \in E'_1$  means that it is transportable between the two end nodes  $S'_i$  and  $S'_j$ ;  $w'_2$  is weights defined on every edge  $e \in E'_2$  as the logistic fee per unit goods between the two ends of  $e$ . If  $(S'_i, S'_j) \notin E'_1$ , then define  $w'_2(S'_i, S'_j) = \infty$ . Denote  $w'_2(S'_i, S'_j)$  be the lowest logistic cost between  $S'_i$  and  $S'_j$ , which can be calculated by,

$$w'_2(S'_i, S'_j) = \min \left\{ \sum_{e \in \Gamma} w'_2(e) \mid \Gamma \text{ is a path between } S'_i \text{ and } S'_j \in S' \right\} \quad (2)$$

**Remark 2.2** The commercial alliance relationship between the auctioneer and each supplier means that the auctioneer is also as a retailer of every supplier in . Besides, the auctioneer can also make some kinds of selling recommendation for them and can integrate the supply resource for them. On the other hand, each supplier in will give the auctioneer some kind price discount and auction commission.

Below, we suppose that there are  $m$  manufacturers expressed by  $M = \{M_1, \dots, M_m\}$  and  $n$  suppliers expressed by  $S = \{S_1, \dots, S_n\}$ .

### Demand Auction Model with Resource Pooling (DAMRP)

The demand auction model with demand resource pooling is for the market where the supply is much larger than the demand. In this situation, the auctioneer would collect the demands from the manufacturers and conduct a reverse auction with the suppliers to help procure the goods/products for the manufacturers. Usually, the more demand the auctioneer collects, the more commission from the manufacturers and more discount from the suppliers. So, the auctioneer should not only have some information about the manufacturer's willingness to pay from industry practices but also collect demand from its demand resource pooling network consisting of many manufacturers. In the demand auction model, the buyers consist of the all suppliers, the sellers are the manufacturers who hold demands and auction agent of both is the auctioneer.

Let  $G_{M'} = (M', E'_1, w'_1)$  be the demand resource pooling network of the auctioneer, where  $M' \in M$ . Without loss of generality, suppose the auctioneer allocates the same spots as that of  $M'_1$ . Denote that  $(P'_{M'_i}, D_{M'_i})$ ,  $i = 1, \dots, m'$  are the demand of manufacturer  $i$ ,  $i = 1, \dots, m'$  submitted to its auctioneer. Let  $P'_{M'_i}$  satisfies  $P'_{M'_i} = P_{M'_i} + w'_1(M'_1, M'_i)$ .

As an alliance or membership with auctioneer, the manufacturer  $M'_i$ ,  $i = 1, \dots, m'$  can get  $\alpha$  discount from the

auctioneer which usually satisfies  $\alpha \leq 5\%$ . Let  $P_{M'_i} = \alpha P'_{M'_i}$  for  $i = 1, \dots, m'$ . Let  $(P_{M_i}, D_{M_i})$  denote the demand of the other  $m - m'$  manufacturers submitted to its auctioneer. Suppose the bidding price  $P_{M_i}$  for all  $i = m' + 1, \dots, m$  includes the logistic fee per unit between the spot of manufacturer  $M_i$  and location of the auctioneer, i.e., the spot where  $M'_1$  locates. We denote  $(P_{M_i}, D_{M_i})$  as the actual demands of  $m$  manufacturers and expressed below:

$$(P_{M_i}, D_{M_i}) = \begin{cases} (P_{M'_i}, D_{M'_i}), & i = 1, \dots, m', \\ (P_{M_i}, D_{M_i}), & i = m' + 1, \dots, m. \end{cases} \quad (3)$$

So the total demand is  $D = \sum_{i=1}^m D_{M_i}$  and the average price

$$\text{is } \bar{P}_M = \sum_{i=1}^m \left( \frac{D_{M_i}}{D} \right) P_{M_i}.$$

Note that  $(P_{S_j}, D)$  is the bidding price of supplier  $j$ ,  $j = 1, \dots, n$  for the total demand  $D$ . The bidding price  $P_{S_j}$  for any  $j$  includes the logistic fee per unit between the spot of supplier  $S_j$  and location of the auctioneer, i.e., the spot where  $M_1$  or  $M'_1$  locates.

The auctioneer auctions the total demand  $D$  with the average price as the highest purchasing price, which serves as a reservation price in the reverse auction:

$$\bar{P}_M = \sum_{i=1}^m \left( \frac{D_{M_i}}{D} \right) P_{M_i} \quad (4)$$

**Remark 2.3** In the model (DAMRP), the demand resource pooling network has the following characteristics: (1) to guarantee the auctioneer's lowest bidding demand; (2) to be more efficient for the inside manufacturers; (3) the network will grow bigger because of the information asymmetry between manufacturers and suppliers.

### Supply Auction Model with Resource Pooling (SAMRP)

The supply auction model suits for the situation that the supply resource is short and there is serious information asymmetry between the sellers and buyers. Thus, the auctioneer should not only have some information of supplier's bidding from auction market but also integrate the supply resources from its supply resource pooling network consisting of the suppliers. Similar to demand auction model, the buyers consist of the all manufacturers, the sellers are the suppliers who hold resources. The auctioneer conducts the auction on the suppliers' behalf, and collects commission from them.

Let  $G_{S'} = (S', E'_2, w'_2)$  be the supply resource pooling network of the auctioneer, where  $S' \subseteq S$ . Without loss of generality, suppose the auctioneer occupies the same spot as that of  $S'_1$ . Denote that  $(P'_{S'_j}, Q_{S'_j})$ ,  $j = 1, \dots, n'$  are the supply of supplier  $j$ ,  $j = 1, \dots, n'$  submitted to its auctioneer. Let  $P'_{S'_j}$  satisfies  $P'_{S'_j} = P'_{S'_1} + w'_2(S'_1, S'_j)$ .

As an alliance with the auctioneer, the supplier  $S'_j$ ,  $j = 1, \dots, n'$  will pay  $\beta$  percent as an appreciation to the auctioneer, usually with  $\beta \leq 5\%$ . Let  $P'_{S'_j} = (1 + \beta)P'_{S'_1}$  for  $j = 1, \dots, n'$ . Let  $\{(P_{S_j}, Q_{S_j}), j = n'+1, \dots, n\}$  denote the supply of other  $n - n'$  suppliers submitted to its auctioneer. Suppose the bidding price  $P_{S_j}$  for any  $j = n'+1, \dots, n$  includes the logistic fee per unit between the spot of supplier  $S_j$  and location of the auctioneer, i.e., the spot for  $S'_1$ . Let  $(P_{S_j}, Q_{S_j})$  denote the actual supply of the suppliers as expressed below:

$$(P_{S_j}, Q_{S_j}) = \begin{cases} (P'_{S'_j}, Q_{S'_j}) & j = 1, \dots, n'; \\ (P_{S_j}, Q_{S_j}) & j = n'+1, \dots, n. \end{cases} \quad (5)$$

So the total supply is  $Q = \sum_{j=1}^n Q_{S_j}$  with the average price of  $\bar{P}_S = \sum_{j=1}^n \left(\frac{Q_{S_j}}{Q}\right) P_{S_j}$ .

Note that  $P_{M_i}$  is the bidding price of manufacturer  $i$ ,  $i = 1, \dots, m$  for the demand  $D_{M_i}$ . The bidding price  $P_{M_i}$  for any  $i$  includes the logistic fee per unit between the spot of manufacturer  $M_i$  and the location of the auctioneer, i.e., the spot where supplier  $S_1$  or  $S'_1$  locates.

The auctioneer auctions the supply resource  $(P_{S_j}, Q_{S_j})$ ,  $j = 1, \dots, n', n'+1, \dots, n$  to the  $m$  manufacturers with bidding  $(P_{M_i}, D_{M_i})$ , where  $i = 1, \dots, m$ .

**Remark 2.4** The supply resource pooling network has the following characteristics: (1) to guarantee the auctioneer's lowest bidding supply; (2) to control the short resource of supply resource pooling network by providing allied suppliers with higher bidding price and service priority, which can also make the network bigger.

### III. Algorithms via Vickrey Auction Mechanism

From the perspective of the auctioneer, we propose two algorithms via Vickrey auction mechanism for the models (DAMRP) and (SAMRP), respectively. We can employ linear programming to obtain the final allocation.

For the model (DAMRP), we propose Algorithm (DAMRP) as follows.

#### Algorithm (DAMRP) via Vickrey Auction mechanism

**Event 0.** Demand collection. (a) From it manufacturers' demands network, the auctioneer can collect the network's demand bidding  $(P'_{M_i}, D_{M_i})$ ,  $i = 1, \dots, m'$ . (b) From auction market, the auctioneer can get the other manufacturers' demand bidding  $(P_{M_i}, D_{M_i})$ , where  $i = m'+1, \dots, m$ . (c) Let  $P_{M'_i} = \alpha P'_{M'_i}$  for  $i = 1, \dots, m'$ . Such,  $P_{M'_i}$  can be calculated by (3) as the actual demand price of manufacturer  $i$ ,  $i = 1, \dots, m', m'+1, \dots, m$ , for its demand  $D_{M_i}$  submitted to its auctioneer.

**Event 1.** Calculate the total demand  $D = \sum_{i=1}^m D_{M_i}$  and the highest purchasing price as follows:  $\bar{P}_M = \sum_{i=1}^m \left(\frac{D_{M_i}}{D}\right) P_{M_i}$ .

**Event 2.** Auction the demand  $D$  with the lowest price  $\bar{P}_M$ .

**Event 3.** Suppose that the final suppliers' bidding is  $(P_{S_j}, D)$ , where  $j = 1, \dots, n$ . Without loss of generalities, let  $P_{S_1} \leq P_{S_2} \leq \dots \leq P_{S_n}$ .

**Event 4.** Make auction decision via Vickrey Auction Mechanism. If  $P_{S_1} \leq P_{S_2} \leq \bar{P}_M$ , supplier  $S_1$  wins and hedging price is  $P_{S_2}$ . Go to Event 5; If  $P_{S_1} \leq \bar{P}_M \leq P_{S_2}$ , supplier  $S_1$  wins and hedging price is  $\bar{P}_M$ . Go to Event 5; Otherwise, remove the manufacturer with the lowest demand price and let  $m = m - 1$ . If  $m \geq 1$ , go to Event 2; else, increase the bidding price of the demand resource pooling network, and repeat the whole process again, i.e. go to Event 0.

**Event 5.** Allocate the supply amount  $D$  to satisfy remaindered manufacturers. Assign the actual demand  $(P_{M_i}, D_{M_i})$  to manufacturer  $M_i$ , where  $i = 1, \dots, m', m'+1, \dots, m$  for its demand  $D_{M_i}$  excepted those manufacturers who are removed during above auction process.

**Remark 3.1** Notice that some manufacturers are removed during the auction process because of their high bidding prices, which promotes them to joint auctioneer's demand

resource pooling network and get alliance discount. Thus, the auctioneer can accumulate large quantities of demand and can get lower hedging price.

**Remark 3.2** The auctioneer's profit can be calculated by

$$\eta \sum_{i=1}^m P_{M_i} D_{M_i} + \eta P_{hedging} D + \sum_{i=1}^m P_{M_i} D_{M_i} - P_{hedging} D - \sum_{i=2}^m D_{M_i} w(S_1, M_i) \quad (8)$$

where  $\eta$  is service fee for the auction process and  $P_{hedging}$  is the hedging price as expressed by:

$$P_{hedging} = \begin{cases} P_{S_2}, & P_{S_1} \leq P_{S_2} \leq \bar{P}_M \\ \bar{P}_M, & P_{S_1} \leq \bar{P}_M \leq P_{S_2} \end{cases} \quad (9)$$

For the model (SAMRP), we propose Algorithm (SAMRP) as follows.

#### **Algorithm (SAMRP) via Vickrey Auction Mechanism**

**Event 0.** Resource collection. (a) From the suppliers' resource network, the auctioneer can collect the network's resources price commitments  $(P'_{S_j}, Q_{S_j})$ ,  $j = 1, \dots, n'$ . (b) From the auction market, the auctioneer can get other supplier's  $(S_j, j = n'+1, \dots, n)$  resource bidding  $(P_{S_j}, Q_{S_j})$ . (c) Let  $P_{S'_j} = (1 + \beta)P'_{S'_j}$  for  $j = 1, \dots, n'$ , where  $\beta$  (usually  $\beta \leq 5\%$ ) is the appreciation rate as the allied member of the supplier resource network from the auctioneer. Such,  $(P_{S_j}, Q_{S_j})$  can be found by (5) as the actual supply bidding of supplier  $S_j$ ,  $j = n'+1, \dots, n$ . Calculate the total supply  $Q = \sum_{j=1}^n Q_{S_j}$

and the average price  $\bar{P}_S = \sum_{j=1}^n \left( \frac{Q_{S_j}}{Q} \right) P_{S_j}$  as the lowest selling price (a reservation price).

**Event 1.** Auction with the resource limitation  $Q = \sum_{j=1}^n Q_{S_j}$

with the average price  $\bar{P}_S = \sum_{j=1}^n \left( \frac{Q_{S_j}}{Q} \right) P_{S_j}$ .

**Event 2.** For the total resource commitment  $(\bar{P}_S, Q)$  suppose that the final manufacturers' bidding  $(P_{M_i}, D_{M_i})$ ,  $i = 1, \dots, m$  with  $P_{M_1} \geq P_{M_2} \geq \dots \geq P_{M_m}$  and  $P_{M_{m+1}} = P_{M_m}$ .

**Event 3.** Let  $Q_{ij}$  be supplied from supplier  $j$  to manufacturer  $i$ ,  $i = 1, \dots, n$  and  $j = 1, \dots, m$ . By Vickrey Auction mechanism, suppose the manufacturer  $M_i$  wins with hedging price is  $P_{M_{i+1}}$  for every  $i = 1, \dots, m$  and  $P_{M_{m+1}} = P_{M_m}$ . Solve the linear programming problem:

$$\begin{aligned} & \text{maximize} && \sum_{i=1}^m \sum_{j=1}^n (P_{M_{i+1}} - P_{S_j} - w(M_i, S_j)) Q_{ij} \\ & \text{subject to} && \sum_{i=1}^m \sum_{j=1}^n (P_{M_{i+1}} - P_{S_j} - w(M_i, S_j)) Q_{ij} \geq 0 \\ & && \sum_{j=1}^n Q_{ij} \leq D_{M_i}, \quad i = 1, \dots, m \\ & && \sum_{i=1}^m Q_{ij} = Q_{S_j}, \quad j = 1, \dots, n \\ & && P_{M_i} \geq \bar{P}_S, \quad i = 1, \dots, m \\ & && Q_{ij} \geq 0, \quad i = 1, \dots, m, \quad j = 1, \dots, n. \end{aligned} \quad (10)$$

**Event 4.** Make auction decision. If linear programming (1) has optimal solution  $Q_{ij}$ ,  $i = 1, \dots, n$  and  $j = 1, \dots, m$ .

Manufacturer  $M_i$  wins with the hedging price is  $P_{M_{i+1}}$  and amount  $D_{M_i}$ ,  $i = 1, \dots, m$ . Go to Event 5; Otherwise, remove the manufacturer with lowest bidding price and let  $m = m - 1$ . If  $m \geq 1$ , then go to Event 3, else remove the supplier with highest price and let  $n = n - 1$ . If  $n \geq 1$ , then go to Event 1; else, based on the auction information, decrease the bidding price of supply resource pooling network, and start all over again, i.e. go to Event 0.

**Event 5.** Allocate the supply amount  $Q_{ij}$  from supplier  $j$  to manufacturer  $i$ ,  $i = 1, \dots, n$ ,  $j = 1, \dots, m$ .

**Remark 3.3** The auction profit can be calculated by above optimal value

$$\eta \sum_{i=1}^m P_{M_{i+1}} D_{M_i} + \eta \sum_{i=1}^m P_{S_j} Q_{S_j} + \sum_{i=1}^m \sum_{j=1}^n (P_{M_{i+1}} - P_{S_j} - w(M_i, S_j)) Q_{ij} \quad (11)$$

For the model (DAMRP), we propose Algorithm (DAMRP) as below.

## **IV. Incentive Compatibility and Walrasian Equilibrium**

### **Incentive Compatibility**

In the general auction model, we know that Vickrey auction is incentive compatible [7]. That is to say that the buyer is satisfied by the transaction that the buyer with the highest bid wins the resource at the price of the 2nd highest bid. In our proposed models, we apply the idea of Vickrey auction mechanism. We can prove that the proposed auction processes with resource pooling for supply chain management are also incentive compatible, i.e. all of the auction winners are satisfied by the transaction that manufacturer winners pay less than they expected and supplier winners get more than they submitted to the auctioneer.

**Theorem 4.1** The two auction models (DAMRP), and (SAMRP) with algorithms (DAMRP) and (SAMRP) respectively, are incentive compatible.

## Walrasian Equilibrium

Intuitively, Walrasian equilibrium is a vector of price and allocation matrix such that (i) all auction winners are satisfied with the corresponding allocation, and (ii) the market clears or the price of non-allocated supply is zero. In the proposed auction models with resources pooling for supply chain management, we define Walrasian equilibrium for our models respectively as follows.

### Definition 4.1 (Walrasian Equilibrium for Auction Model (DAMRP))

For the proposed auction model (DAMRP) with algorithm (DAMRP) via Vickrey Auction Mechanism, we define a Walrasian equilibrium is a hedging supply series  $\{(P_{S_j}, Q_{S_j}), j=1, \dots, n\}$  and demand series  $\{(P_{M_i}, D_{M_i}), i=1, \dots, m\}$  allocation matrix  $Q_{ij}$  from supplier  $S_j$  to manufacturer  $M_i$ ,  $i=1, \dots, n$  and  $j=1, \dots, m$  such that (i) each supply winner is satisfied with the hedging price and each manufacturer winner is satisfied with the demand allocated, (ii) The auctioneer's utility is maximized by the corresponding allocation. (iii) Final sum of bidding demands can be produced by supplies and sold out to the manufacturer winners.

### Definition 4.2 (Walrasian Equilibrium for Auction Model (SAMRP))

For the proposed auction model (SAMRP) with algorithm (SAMRP) via Vickrey Auction Mechanism, we define a Walrasian equilibrium is a hedging demand series  $\{(P_{M_i}, D_{M_i}), i=1, \dots, m\}$ , supply series  $\{(P_{S_j}, Q_{S_j}), j=1, \dots, n\}$  and allocation matrix  $Q_{ij}$  from supplier  $S_j$  to manufacturer  $M_i$ ,  $i=1, \dots, n$  and  $j=1, \dots, m$  such that (i) each manufacturer winner is satisfied with the hedging price and amounts allocated, (ii) The auctioneer's utility is maximized by the corresponding allocation. (iii) Final sum of bidding supply can be sold out to the manufacturer winners.

**Theorem 4.2** The auction model (DAMRP) with algorithm (DAMRP) has Walrasian Equilibrium.

**Theorem 4.3** The auction model (SAMRP) with algorithm (SAMRP) has Walrasian Equilibrium.

## V. Conclusion and Further Research

In the paper, we propose two theoretic models and corresponding algorithms of supply chain auctions with

resource pooling according using the Vickrey auction principle, which achieves three functions: price mining, dynamic supply chain construction and resources integrating. Besides, these proposed models are much closer to practical settings and may have potential applications in supply chain management.

There are many possible directions for future theoretical studies of these models. For example, the concept and existence about other kinds of equilibrium, the optimal allocations (corresponding to linear and duality approach), complexity analysis, etc.

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